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Optimal Taxation with Cournot Oligopoly

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Optimal Taxation with Cournot Oligopoly*

Leslie J. Reinhorn

Abstract

This paper studies optimal linear taxation in a general equilibrium model with Cournot oligopoly. The main result is the following. With imperfect competition the tendency toward “inverse elasticities” tax rules will be weakened and may even be reversed. That is, an upward sloping relationship may exist between an industry’s optimal tax rate and its own-price elasticity of demand, unlike the perfectly competitive case.

KEYWORDS: Optimal taxation, Cournot oligopoly

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1 Introduction

One of the central results in public economics is the Ramsey (1927) optimal tax rule. This paper will show that the structure of optimal taxes implied by the Ramsey rule is highly dependent upon the assumption of perfectly competitive producer behavior. In the perfectly competitive case optimal tax rates tend to follow an inverse elasticities rule: smaller own-price elasticities are generally associated with larger tax rates. When firms are price setters this will be weakened, and may even be reversed: the relationship between the optimal tax rate and the own-price elasticity may be upward sloping.¹

This result is based on the following. In the presence of imperfect competition, the government's optimal tax policy has a two-part interpretation:² First, subsidize each industry to completely offset the producer markups. This brings prices back down to marginal costs. Second, use the familiar Diamond and Mirrlees (1971) (hereafter DM) tax rule to raise sufficient revenue to finance the subsidies and also to finance the government's other operations.

The first part of this decomposition works directly against an inverse elasticities tax rule. A smaller elasticity of demand allows imperfectly competitive firms to charge a higher markup, which will be offset with a larger subsidy. If lump sum taxation were available, this would be the end of the story. The government would use an inverse elasticities rule for *subsidies* and it would pay for this with the lump sum tax. But in a second best world, the revenue must come from distortionary taxation — the second part of the decomposition — and this generates the standard inverse elasticities rule.

Thus the two parts push the optimal tax rate in opposite directions. This establishes that the inverse elasticities rule will be weakened, but will it be reversed? The key observation here is that the DM formula includes cross price effects while the producer markups do not. Specifically, the government internalizes all economic repercussions from each of its taxes because it is concerned with overall economic welfare. By contrast, each firm is only concerned with the profits it can extract from the industry — or few industries — in which it operates. This indicates that the first part of the decomposition (the subsidy) is likely to be more elasticity sensitive than the second part (the DM formula). The net result can be a reversal of the inverse elasticities rule. This will be illustrated with an example in section 4.

The first part of the tax decomposition may appear to reward high markups

¹This result is apparently new. The literature on taxation in imperfectly competitive economies has generally focused on other issues. Myles (1989) analyzes the impact of optimal commodity taxes on compensated demand (the index of discouragement), under the restriction that profits are not taxed. Auerbach and Hines (2002, section 6) and Myles (1995, chapter 11) review the literature.

²Guesnerie and Laffont (1978, p. 450) and Stiglitz (1987, p. 1036) briefly mention this two-part interpretation, but they do not address the implications for inverse elasticities rules.

with lower tax rates. However, it is the demand elasticities that drive both.

The paper uses a model of Cournot competition with homogeneous goods and a fixed number of firms.³ In this setting, the elasticity of demand faced by price setting firms is directly related to the elasticity of demand for the industry as a whole. The former elasticity determines the size of the markups, while the latter enters into the DM formula. This facilitates direct comparison of the two parts of the tax.

One would like to know the extent to which the optimal tax system can correct the oligopolistic distortions. This question has been addressed previously by Myles (1996). He finds that when the government has access to both an ad valorem tax and a specific tax for each industry,⁴ it is possible to eliminate all of the adverse effects from oligopoly and thereby attain the DM second best welfare level. The net outcome — consumer prices and the allocation — is exactly the same as the perfectly competitive DM case. This result is also obtained here.⁵ The reason is that the government is equipped with two independent tax instruments in each industry. In the present case these instruments are an industry-specific commodity tax and profits tax. With two independent instruments for each industry, the government is able to achieve two targets per industry: the consumer price and the level of distributed profits. In particular, the DM consumer prices may be achieved as well as DM's zero dividends — the latter via 100 percent taxation of profits. Presumably redistributive goals might be met if the profits of different industries were taxed at different rates. However, fine tuning of this instrument seems especially impractical.⁶

There is one quirk that may hinder the implementation of the optimal policy. Recall that a monopolist will never operate where the elasticity of demand is less than one. An analogous bound holds for Cournot oligopolists. As indicated above, when the government has access to two instruments per industry, in principle it can achieve the DM consumer prices and allocation. However, the DM optimum imposes no a priori restriction on the elasticity of demand, so it may fail to satisfy the Cournot bound. In this case the DM outcome will not be implementable. It turns out that this problem cannot arise if all commodity taxes are specific, but for ad valorem taxes it is a possibility. In previous results, tax theorists have generally found that ad valorem taxation dominates specific taxation in the presence of imperfect competition.⁷ The

³The argument extends to other models with producer markups (Reinhorn, 2003).

⁴Recall that a specific tax is one that is levied on the number of units sold, while an ad valorem tax is levied on the value of sales.

⁵See also Guesnerie and Laffont (1978, theorem 5).

⁶Appendix D briefly considers the other extreme where profits are not taxed at all. It should be noted that profits may also be present in the perfectly competitive case — e.g., Mirrlees (1972), Munk (1978), Stiglitz and Dasgupta (1971). The distinguishing feature of imperfect competition is markups, not profits.

⁷See Suits and Musgrave (1953) for the classic result in this area. For more recent

different results are a consequence of the 100 percent profits tax here. This suggests that when the government must choose between two similar instruments, the optimal choice may be affected by the set of other instruments already in use.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the optimal tax problem and presents the main theoretical results. Section 4 uses an example to illustrate the results. Section 5 addresses the implementation problem that may arise with ad valorem taxes, and shows that this problem does not arise with specific taxes. Section 6 concludes the paper.

2 Model

The economy has I industries labeled $i = 1, \dots, I$. Industry i contains a finite number of firms, n_i , each of which produces the same homogeneous good. There is a continuum of identical consumers of mass one.⁸ Their utility function is $U(\ell, Q_1, \dots, Q_I)$, where ℓ is consumption of leisure and Q_i is consumption of good i . Each consumer is endowed with L units of time and zero units of good $i \geq 1$.

Labor is the only factor of production. Each of the n_i firms in industry i has the same cost function $C_i(q)$, where q is the firm's output and the costs are measured in units of labor. Fixed costs in industry i are $C_i(0^+) := \lim_{q \downarrow 0} C_i(q)$. Total output in the industry is $Q_i = \sum_{j=1}^{n_i} q_{ij}$ where q_{ij} is the output of firm j in industry i .

The government uses an exogenous quantity of labor, g , for public production. This is taken as exogenous in order to focus exclusively on tax policy. Thus, public goods do not appear in the consumers' utility function since they never vary. To finance its operations, the government taxes the consumers' labor income at rate t_0 and taxes industry i at ad valorem rate t_i . (Section 5 considers specific taxes.) Profits in industry i are taxed at rate t_i^π . Lump sum transfers are ruled out.⁹

Producer prices are denoted p_0, p_1, \dots, p_I where p_0 is the price of labor. Consumer prices are P_0, P_1, \dots, P_I . These are related by $P_0 = (1 - t_0)p_0$ and $P_i = (1 + t_i)p_i$ for $i \geq 1$.

treatments, see Delipalla and Keen (1992) and Skeath and Trandel (1994), though the latter paper is subject to the usual criticisms associated with the use of a partial equilibrium framework for the study of a tax problem. The first result in favor of ad valorem appears to be Wicksell (1959) (originally published in 1896) who considered the case of monopoly under partial equilibrium.

⁸The extension to heterogeneous consumers is discussed in the conclusion.

⁹In models with heterogeneous consumers it is reasonable to include a poll tax (or subsidy). But with identical consumers, a poll tax would allow the government to attain the first best.

2.1 Consumers

Each consumer chooses $(\ell, Q_1, \dots, Q_I) \geq \mathbf{0}$ to

$$\begin{aligned} & \text{maximize} \quad U(\ell, Q_1, \dots, Q_I) \\ & \text{subject to} \quad \sum_{i=1}^I P_i Q_i \leq P_0(L - \ell) + \pi_{\text{net}} \end{aligned}$$

where π_{net} is profits net of taxes. Let $M := P_0L + \pi_{\text{net}}$. Then the budget constraint can be written $P_0\ell + \sum_{i=1}^I P_i Q_i \leq M$. The solution yields consumption functions $\ell(\mathbf{P}, M)$ and $Q_i(\mathbf{P}, M)$ for $i \geq 1$. Indirect utility is $V(\mathbf{P}, M) := U(\ell(\mathbf{P}, M), \mathbf{Q}(\mathbf{P}, M))$. Since the mass of consumers is one, these apply both at the individual level and in the aggregate.

2.2 Firms

Within each industry, firms are profit maximizing Cournot oligopolists.¹⁰ Profits for a typical firm in industry i are

$$p_i \cdot [Q_i(\mathbf{P}, M) - \hat{Q}_i] - p_0 C_i(Q_i(\mathbf{P}, M) - \hat{Q}_i) \quad (1)$$

where $\hat{Q}_i < Q_i$ is the output of all other firms in industry i . Each firm treats the values of \hat{Q}_i , M , t_i , p_0 , and $P_{i'}$ ($i' \neq i$) as parameters and controls the producer price p_i , choosing it to maximize profits.

Note that these Cournot oligopolists choose price rather than quantity. But with downward sloping demand curves, the two are equivalent. The key point is that firms take \hat{Q}_i as given, regardless of whether price or quantity is the choice variable.

The firms' profit criterion in (1) is somewhat myopic. Gordon (2003) observes that shareholders want the value of their *portfolios* maximized. In particular, they want the manager of each firm to internalize the effect of his decisions on the value of their entire portfolio (and more generally, on the value of their expected utility). If they can exercise influence over managers' decisions, this may have a significant impact on firms' objectives. This topic is still open for debate. For now, at least, the myopic criterion in (1) seems to be the predominant view.

The first order condition for an interior maximum to (1) is

$$(Q_i - \hat{Q}_i) + P_i \partial Q_i / \partial P_i - (1 + t_i) p_0 C'_i \partial Q_i / \partial P_i = 0.$$

The condition for a symmetric Cournot equilibrium in industry i is

$$Q_i(\mathbf{P}, M) / n_i + P_i \partial Q_i / \partial P_i - (1 + t_i) p_0 C'_i(Q_i(\mathbf{P}, M) / n_i) \cdot \partial Q_i / \partial P_i = 0 \quad (2)$$

¹⁰The extension to conjectural variations is straightforward.

which incorporates the consistency condition $\hat{Q}_i = (n_i - 1)Q_i/n_i$. Equation (2) must hold simultaneously for all industries.

Let $\epsilon_{ii} > 0$ be the own-price elasticity of demand. Then (2) has the familiar form

$$\frac{p_i - p_0 C'_i}{p_i} = \frac{1}{n_i \epsilon_{ii}} \quad (3)$$

which states that the Lerner index is negatively related to both the elasticity and the number of firms. Since the left hand side of (3) is less than one, $\epsilon_{ii} > 1/n_i$ in equilibrium. This is the elasticity bound for Cournot oligopoly, which generalizes the monopoly case.

2.3 Equilibrium

An equilibrium is a vector

$$(\bar{t}_0, \bar{t}_1, \dots, \bar{t}_I, \bar{t}_1^\pi, \dots, \bar{t}_I^\pi, \bar{g}, \bar{\ell}, \bar{Q}_1, \dots, \bar{Q}_I, \bar{p}_0, \bar{P}_0, \dots, \bar{P}_I, \bar{M})$$

of the labor income tax rate, the sales tax rates, the tax rates on profits, government expenditure, consumption quantities, the producer price for labor, consumer prices, and lump sum income that satisfies the following conditions:

$$\bar{\ell} = \ell(\bar{\mathbf{P}}, \bar{M}), \quad \bar{Q}_i = Q_i(\bar{\mathbf{P}}, \bar{M}) \quad i \geq 1 \quad (4)$$

$$\bar{Q}_i/n_i + \bar{P}_i \partial Q_i / \partial P_i - (1 + \bar{t}_i) \bar{p}_0 C'_i(\bar{Q}_i/n_i) \cdot \partial Q_i / \partial P_i = 0 \quad i \geq 1 \quad (5)$$

$$\bar{P}_0 = (1 - \bar{t}_0) \bar{p}_0 \quad (6)$$

$$\bar{M} = \bar{P}_0 L + \sum_{i=1}^I (1 - \bar{t}_i^\pi) [\bar{P}_i \bar{Q}_i / (1 + \bar{t}_i) - \bar{p}_0 n_i C_i(\bar{Q}_i/n_i)] \quad (7)$$

$$\bar{g} + \bar{\ell} + \sum_{i=1}^I n_i C_i(\bar{Q}_i/n_i) \leq L \quad (8)$$

where $\partial Q_i / \partial P_i$ in (5) is evaluated at $(\bar{\mathbf{P}}, \bar{M})$.

Equation (4) states that the quantities, prices, and income are consistent with utility maximization. Then (5) is the condition for a symmetric Cournot equilibrium, while (6) and (7) are just the definitions of P_0 and M . Finally, (8) is the resource constraint. By Walras' Law, when all these conditions are met the government automatically satisfies its budget constraint.

3 Optimal taxation

The government's goal is to choose a feasible tax policy that maximizes the welfare of a representative consumer while financing a given level of government spending g . The class of feasible policies will be those which tax away all economic profits, $\mathbf{t}^\pi = \mathbf{1}$. This is optimal when all consumers are identical since it provides a non-distortionary source of tax revenue (which, under normal circumstances, does not exceed the government's overall revenue requirement). With heterogeneous consumers, adjustments to \mathbf{t}^π may achieve distributive goals. However, as indicated in the introduction, this would be highly impractical. Appendix D briefly considers the other extreme, no taxation of profits. The analysis below focuses on the sensitivity of the optimal tax rate t_i to the own-price elasticity ϵ_{ii} .

The formal statement of the government's problem is identical to the DM perfectly competitive case. That is, the government's goal is to find tax rates t_0, t_1, \dots, t_I that support the solution to the following indirect utility maximization problem: Choose $\mathbf{P} \geq \mathbf{0}$ to

$$\begin{aligned} & \text{maximize} && V(\mathbf{P}, M) \\ & \text{subject to} && g + \ell(\mathbf{P}, M) + \sum_{i=1}^I n_i C_i(Q_i(\mathbf{P}, M)/n_i) \leq L. \end{aligned} \quad (9)$$

Here $M = P_0 L$ since $\mathbf{t}^\pi = \mathbf{1}$. Let $\eta \geq 0$ be the Lagrange multiplier for (9). It measures the marginal disutility of an increase in the government's spending requirement g . If $\mathbf{P} \gg \mathbf{0}$, the first order conditions are (9) and

$$\partial V / \partial P_j = \eta \left[\partial \ell / \partial P_j + \sum_{i=1}^I C'_i \partial Q_i / \partial P_j \right] \quad j \geq 1 \quad (10)$$

$$\frac{\partial}{\partial P_0} V(\mathbf{P}, P_0 L) = \eta \frac{\partial}{\partial P_0} \left[\ell(\mathbf{P}, P_0 L) + \sum_{i=1}^I n_i C_i(Q_i(\mathbf{P}, P_0 L)/n_i) \right]. \quad (11)$$

By homogeneity, (11) is redundant when (10) holds for all $j \geq 1$. Once an optimal \mathbf{P} is determined and suitably normalized, the optimal tax rates are found by solving (5) and (6), with p_0 also available for normalization. Two technical issues are addressed in appendices. Appendix B proves that the optimization problem has a solution. Appendix C establishes the necessity of the first order conditions.

In raw form, the first order conditions do not offer much insight. The following manipulations will yield a form that links the optimal policy to elasticities.

Substitute Roy's identity, $\partial V/\partial P_j = -Q_j V_M$ where $V_M := \partial V/\partial M$, into (10) to get

$$-Q_j V_M = \eta \left[\partial \ell / \partial P_j + \sum_{i=1}^I C'_i \partial Q_i / \partial P_j \right] \quad j \geq 1. \quad (12)$$

Since the consumers' budget constraint holds as an identity in (\mathbf{P}, M) , differentiation with respect to P_j yields

$$0 = P_0 \partial \ell / \partial P_j + Q_j + \sum_{i=1}^I P_i \partial Q_i / \partial P_j \quad j \geq 1. \quad (13)$$

Multiply (12) by P_0 , (13) by η , subtract, and rearrange to get

$$\frac{\eta - P_0 V_M}{\eta} = - \sum_{i=1}^I \frac{P_i - P_0 C'_i}{P_i} \cdot \frac{P_i Q_i}{P_j Q_j} \cdot \frac{P_j}{Q_i} \frac{\partial Q_i}{\partial P_j} \quad j \geq 1. \quad (14)$$

Let

$$\tilde{\epsilon}_{ij} := - \frac{P_i Q_i}{P_j Q_j} \cdot \frac{P_j}{Q_i} \frac{\partial Q_i}{\partial P_j} \quad i \geq 0, j \geq 0$$

where $Q_0 := \ell$. The $\tilde{\epsilon}_{ij}$ terms are expenditure weighted elasticities of demand; $\tilde{\epsilon}_{jj}$ is just the own-price elasticity. Note that (13) imposes an adding-up constraint: $\sum_{i=0}^I \tilde{\epsilon}_{ij} = 1$ for all j . Equation (14) can be written as

$$\sum_{i=1}^I \frac{P_i - P_0 C'_i}{P_i} \tilde{\epsilon}_{ij} = \frac{\eta - P_0 V_M}{\eta} \quad j \geq 1, \quad \text{or} \quad \mathbf{z}^T \tilde{E} = \mathbf{1}^T \quad (15)$$

where the I -vector \mathbf{z} has entries $z_i = x_i/y$ with x_i equal to the gross markup $1 - P_0 C'_i/P_i$ and $y = 1 - P_0 V_M/\eta$, a measure of marginal excess burden. The $I \times I$ matrix \tilde{E} has $\tilde{\epsilon}_{ij}$ in row i , column j . The superscript T indicates the transpose operator. The gross markup x_i combines the producer markup and the tax.

As a benchmark, consider the outcome if optimal lump sum taxation is available. Since the consumers' first order conditions yield $P_0 V_M = \partial U/\partial \ell$ while the envelope theorem and (9) yield $\eta = -\partial V^*/\partial g$, y is the proportional difference between the social marginal value of the factor of production and its private marginal value. At the first best, y must be zero. Then from (15), marginal cost pricing is optimal, $P_i = P_0 C'_i$. The government achieves this with subsidies that offset the producer markups. This is the inverse elasticities rule for *subsidies* that was discussed in the introduction. Return now to the more interesting case when lump sum taxation is limited to the profits tax.

3.1 Inverse elasticities rules and reversals

This section addresses the consequences of a marginal change in the own-price elasticity $\tilde{\epsilon}_{ii}$. First I analyze the effect on the optimal gross markup x_i . Then I use this to address the effect on the optimal tax rate t_i . Generally, one expects a downward sloping relationship between x_i and $\tilde{\epsilon}_{ii}$. This would be the case in partial equilibrium, so if general equilibrium interactions are not too pervasive the result should also apply here. The analysis treats $\tilde{\epsilon}_{ii}$ as exogenous and assumes all other entries in \tilde{E} are independent of $\tilde{\epsilon}_{ii}$: $\partial \tilde{\epsilon}_{jk} / \partial \tilde{\epsilon}_{ii} = 0$ for all $(j, k) \neq (i, i)$ where $ijk \neq 0$. This is a strong assumption and, at best, an approximation. However, even with these general equilibrium effects shut down, the familiar inverse elasticities rule $\partial x_i / \partial \tilde{\epsilon}_{ii} < 0$ is not an immediate consequence. In the general case, without the elasticity independence assumption, the range of possibilities is even greater. One still expects an inverse elasticities rule, but caution is warranted.

With the above assumptions, differentiation of (15) with respect to $\tilde{\epsilon}_{ii}$ yields

$$\frac{\partial \mathbf{z}^T}{\partial \tilde{\epsilon}_{ii}} \tilde{E} = -z_i \mathbf{u}_i^T \quad i \geq 1$$

where \mathbf{u}_i is the unit vector with 1 in row i . Since \tilde{E} will be generically non-singular,

$$\frac{\partial \mathbf{z}^T}{\partial \tilde{\epsilon}_{ii}} = -z_i \mathbf{u}_i^T \tilde{E}^{-1} \quad i \geq 1.$$

Post-multiply by \mathbf{u}_i and rearrange to get

$$\frac{\tilde{\epsilon}_{ii}}{z_i} \frac{\partial z_i}{\partial \tilde{\epsilon}_{ii}} = -\tilde{\epsilon}_{ii} (\tilde{E}^{-1})_{ii} = -(\tilde{E})_{ii} (\tilde{E}^{-1})_{ii} \quad i \geq 1 \quad (16)$$

where $(\cdot)_{ij}$ denotes the entry in row i , column j of the corresponding matrix. Observe that $\tilde{E} = \text{diag}(P_j Q_j) E \text{diag}(1/P_j Q_j)$ where diag indicates the $I \times I$ diagonal matrix with main diagonal entries as indicated and where E is the $I \times I$ matrix with unweighted (signed) elasticity ϵ_{ij} in row i , column j . Hence $(\tilde{E}^{-1})_{ii} = (E^{-1})_{ii}$. And since $\tilde{\epsilon}_{ii} = \epsilon_{ii}$, (16) becomes

$$\frac{\partial \log z_i}{\partial \log \epsilon_{ii}} = -(E)_{ii} (E^{-1})_{ii}, \quad \text{or} \quad \frac{\partial \log x_i}{\partial \log \epsilon_{ii}} = \frac{\partial \log y}{\partial \log \epsilon_{ii}} - (E)_{ii} (E^{-1})_{ii} \quad (17)$$

for $i \geq 1$, where the second equality uses the definition $z_i := x_i/y$.

3.1.1 Lemma. *Assume ϵ_{ii} is exogenous. Assume all of the other expenditure weighted elasticities in the matrix \tilde{E} are independent of ϵ_{ii} . If taxes are optimal, (17) must be satisfied where $x_i = (P_i - P_0 C'_i)/P_i$ is the gross markup (producer markup and tax) and $y = (\eta - P_0 V_M)/\eta$ is a measure of marginal*

excess burden. A negative relationship exists between the industry i markup and the own-price elasticity if $(E)_{ii}(E^{-1})_{ii} > \partial \log y / \partial \log \epsilon_{ii}$. I.e., this is the condition for a weak inverse elasticities rule. If y is independent of ϵ_{ii} , the condition for a pure inverse elasticities rule is $(E)_{ii}(E^{-1})_{ii} = 1$, which will be satisfied in the absence of cross-price effects between good i and the other numbered goods (though cross-price effects with leisure may be present).

3.1.2 Remark. “Inverse elasticities” here refers to a downward sloping relationship between the gross markup and the own-price elasticity, for a given industry. It does not refer to a cross-industry relationship in which industries with larger elasticities have smaller markups. From (15), $x_i = y \mathbf{1}^T \tilde{E}^{-1} \mathbf{u}_i = y [P_1 Q_1 \cdots P_I Q_I] E^{-1} \mathbf{u}_i / P_i Q_i$. This formula does not lead to a simple condition under which $\epsilon_{ii} > \epsilon_{jj}$ would imply $x_i < x_j$. The literature on uniform taxation (e.g., Besley and Jewitt, 1995, and Deaton, 1979) suggests that cross-industry comparisons will not lead to general results. ■

3.1.3 Remark. The condition in the lemma that generates the pure inverse elasticities rule is highly restrictive. One would be ill-advised to use this rule for policy. On the other hand, the weak rule is a fairly standard policy prescription in public economics. It may come as a surprise that the weak rule is not an automatic result, just a tendency. However, this is due to general equilibrium effects. The government realizes that in order to raise revenue to pay for g it must withdraw real physical resources from the economy, as stated in the resource constraint (9). If demand in an industry becomes less elastic, a higher tax rate can now be more effective at generating revenue, but physical resources are now more difficult to withdraw. So a higher tax rate does not provide much relief for the government to satisfy the resource constraint, and the optimal response is not immediately clear. ■

The lemma addressed the sensitivity of the gross markup to changes in the own-price elasticity. The main concern, however, is the sensitivity of the tax rate:

3.1.4 Proposition. (a) Normalize $P_0 = p_0 = 1$. The optimal ad valorem tax rate in industry i responds positively to a marginal increase in the own-price elasticity ϵ_{ii} when $-\partial \log x_i / \partial \log \epsilon_{ii} < (p_i - C'_i) / (P_i - C'_i)$ where x_i is the gross markup. I.e., when this is satisfied, the inverse elasticities tax rule is reversed.¹¹ (b) Also assume that ϵ_{ii} is exogenous and all other expenditure weighted elasticities in \tilde{E} are independent of ϵ_{ii} . Then a reversal occurs when $(E)_{ii}(E^{-1})_{ii} - \partial \log y / \partial \log \epsilon_{ii} < (p_i - C'_i) / (P_i - C'_i)$ where $y = (\eta - P_0 V_M) / \eta$ is a measure of marginal excess burden.

¹¹If marginal cost is constant, the analogous condition for a specific tax is the following: $-\partial \log x_i / \partial \log \epsilon_{ii} < (p_i / P_i)(p_i - C'_i) / (P_i - C'_i)$.

Proof. Recall from (3) that the producer markup satisfies $1 - p_0 C'_i / p_i = 1 / (n_i \epsilon_{ii})$. Also, the gross markup is given by $x_i = 1 - P_0 C'_i / P_i$. Hence, the ad valorem tax is

$$1 + t_i = \frac{P_i}{p_i} = \frac{P_0 C'_i / (1 - x_i)}{p_0 C'_i / [1 - 1 / (n_i \epsilon_{ii})]} = \frac{P_0}{p_0} \frac{1 - 1 / (n_i \epsilon_{ii})}{1 - x_i}.$$

Under the normalization $P_0 = p_0 = 1$, this yields

$$\frac{\partial \log(1 + t_i)}{\partial \log \epsilon_{ii}} = \frac{1}{n_i \epsilon_{ii} - 1} + \frac{x_i}{1 - x_i} \frac{\epsilon_{ii}}{x_i} \frac{\partial x_i}{\partial \log \epsilon_{ii}} = \frac{p_i - C'_i}{C'_i} + \frac{P_i - C'_i}{C'_i} \frac{\epsilon_{ii}}{x_i} \frac{\partial x_i}{\partial \log \epsilon_{ii}}$$

where the last equality follows from (3) and the definition of x_i . The inverse elasticities tax rule is reversed when this expression is positive, i.e., when $(p_i - C'_i) / (P_i - C'_i) > -\partial \log x_i / \partial \log \epsilon_{ii}$. This proves part (a). Now use (17) to prove part (b). ■

3.1.5 Remark. The key insight from the proposition is: the greater is the producer markup, the greater is the scope for the inverse elasticities tax rule to be reversed. More specifically, since the producer markup $p_i - C'_i$ is a measure of the degree of imperfect competition, the condition in part (a) requires the perfectly competitive Ramsey effect $-\partial \log x_i / \partial \log \epsilon_{ii}$ to be dominated by the imperfect competition effect $(p_i - C'_i) / (P_i - C'_i)$. This formalizes the discussion in the introduction regarding the two-part interpretation for the optimal tax policy. Also note that if industry i is a small part of the whole economy, $\partial \log y / \partial \log \epsilon_{ii} \approx 0$. Then the condition in part (b) cannot be satisfied when cross-price effects are absent, i.e., it cannot be satisfied when $(E)_{ii} (E^{-1})_{ii} = 1$ (unless good i is subsidized). A pro-elasticities tax rule is likely to require cross-price effects. ■

3.1.6 Remark. The analysis above depends on the chosen normalizations. Consider the ad valorem tax rate $t_1 = P_1 / p_1 - 1$. Obviously if the normalization for consumer prices is $P_1 = 1$, while that for producer prices is $p_1 = 1$, then t_1 will be quite unaffected by changes in elasticities. The role of normalizations is also addressed by Gaube (2005) in the context of environmental taxation. Here, the normalizations fix the consumer and producer prices in the model's only competitive market, labor. ■

4 Example

The previous section provided conditions under which a pro-elasticities tax rule could be optimal for an imperfectly competitive economy. In particular, the condition in proposition 3.1.4(a) requires the producer markup to be sufficiently large and the Ramsey inverse elasticities effect $\partial \log x_i / \partial \log \epsilon_{ii} < 0$ to

be sufficiently small in magnitude. To explore this further, this section presents a numerical example in which the inverse elasticities tax rule is reversed for some parameter values.

The economy has two industries, $I = 2$. The consumers' utility function is $U(\ell, Q_1, Q_2) = \ell + A Q_1^{\alpha_1} Q_2^{\alpha_2}$ with $A = \alpha_1^{-\alpha_1} \alpha_2^{-\alpha_2}$ and $\alpha_i = (\epsilon_i - 1)/(\epsilon_1 + \epsilon_2 - 1)$.¹² The ϵ_i s are the own-price elasticities. They are restricted to satisfy $\epsilon_i > 1$. This ensures concavity of U . The relationship $\epsilon_i = 1 + \alpha_i/(1 - \alpha_1 - \alpha_2)$ is useful. The solution to the consumers' problem is

$$\begin{aligned} Q_i &= \alpha_i (P_i/P_0)^{-\epsilon_i} (P_j/P_0)^{1-\epsilon_j} \quad \text{for } j = 3 - i \\ \ell &= L - (\alpha_1 + \alpha_2) (P_1/P_0)^{1-\epsilon_1} (P_2/P_0)^{1-\epsilon_2} \end{aligned}$$

and indirect utility is $V = L + (1 - \alpha_1 - \alpha_2) (P_1/P_0)^{1-\epsilon_1} (P_2/P_0)^{1-\epsilon_2}$. Assume L is sufficiently large that $\ell \geq 0$.

The firms' cost functions are affine, $C_i(q) = F_i + c_i q$. Markups are determined by (3):

$$\frac{p_i - p_0 c_i}{p_i} = \frac{1}{n_i \epsilon_i}, \quad \text{or} \quad \frac{p_i}{p_0} = \frac{n_i \epsilon_i c_i}{n_i \epsilon_i - 1}.$$

For these functional forms, the matrix of expenditure weighted elasticities is

$$\tilde{E} = \begin{pmatrix} \epsilon_1 & \epsilon_1 - 1 \\ \epsilon_2 - 1 & \epsilon_2 \end{pmatrix}$$

and hence the optimality condition (15) yields

$$z_1 = z_2 = 1/(\epsilon_1 + \epsilon_2 - 1).$$

Recall that z_i is the ratio of the gross markup x_i to the marginal excess burden y . Thus z_i follows a clean inverse elasticities rule, but the behavior of the markup must be disentangled from the excess burden before reaching any further conclusions.

Also, note the role of general equilibrium effects. These were curtailed in the derivation of (16) and (17) by the independence assumption, $\partial \tilde{\epsilon}_{jk}/\partial \tilde{\epsilon}_{ii} = 0$ for all $(j, k) \neq (i, i)$. But here, $\partial \tilde{\epsilon}_{12}/\partial \tilde{\epsilon}_{11} = 1 = \partial \tilde{\epsilon}_{21}/\partial \tilde{\epsilon}_{22}$. This just reinforces the importance of economy-wide linkages in the optimal tax problem.

In addition to (15), the other condition for the government's optimum is

¹²Although this U does not satisfy assumption 1 in appendix A.1, it is adequately well behaved. The demand curve for good i is smooth, downward sloping, and asymptotic to the price axis.

the resource constraint (9), with equality:

$$\begin{aligned} g + n_1 F_1 + n_2 F_2 &= L - \ell - c_1 Q_1 - c_2 Q_2 \\ &= (\alpha_1 + \alpha_2)(P_1/P_0)^{1-\epsilon_1}(P_2/P_0)^{1-\epsilon_2} - c_1 \alpha_1 (P_1/P_0)^{-\epsilon_1} (P_2/P_0)^{1-\epsilon_2} \\ &\quad - c_2 \alpha_2 (P_1/P_0)^{1-\epsilon_1} (P_2/P_0)^{-\epsilon_2}. \end{aligned}$$

Since $z_1 = z_2$ and since $z_i = x_i/y$, it follows that $P_0 c_1/P_1 = P_0 c_2/P_2$. Replace P_2 with $c_2 P_1/c_1$ in the resource constraint and collect terms to get

$$\frac{P_1}{P_0 c_1} = 1 + \left[\frac{g + n_1 F_1 + n_2 F_2}{\alpha_1 + \alpha_2} c_1^{\epsilon_1-1} c_2^{\epsilon_2-1} \right] \left(\frac{P_1}{P_0 c_1} \right)^{\epsilon_1+\epsilon_2-1}. \quad (18)$$

This equation has a number of properties. It has a solution for $P_1/P_0 c_1$ if and only if $g + n_1 F_1 + n_2 F_2 \leq c_1^{1-\epsilon_1} c_2^{1-\epsilon_2} (\epsilon_1 + \epsilon_2 - 2)^{\epsilon_1+\epsilon_2-1} / (\epsilon_1 + \epsilon_2 - 1)^{\epsilon_1+\epsilon_2}$. Not surprisingly, government purchases plus fixed costs cannot be too large. If this condition holds with equality, the solution is unique. Otherwise the equation has two positive solutions. The smaller of the two is optimal for the government's problem since V is decreasing in P_1/P_0 and P_2/P_0 . Iterations on (18), starting from $P_1/P_0 c_1 = 0$, converge monotonically to the smaller solution. That solution always satisfies $1 \leq P_1/P_0 c_1 \leq 1 + 1/(\epsilon_1 + \epsilon_2 - 2)$.

Observe from (18) that every parameter other than L affects $P_1/P_0 c_1$. The Ramsey prices do not satisfy a simple, clean inverse elasticities rule.

Table 1 and figure 1 show prices and tax rates as the elasticity ϵ_1 is varied. Both p_0 and P_0 are normalized to unity. The results show that the consumer price P_1 (and hence the gross markup $1 - c_1/P_1$) is decreasing in ϵ_1 . This would be the standard inverse elasticities rule in the perfectly competitive case. But here, the specific tax $P_1 - p_1$ and the ad valorem tax $t_1 = (P_1 - p_1)/p_1$ are affected by the producer markup. Both of these tax rates respond inversely for small values of ϵ_1 , but the inverse elasticities rule is reversed for larger values.

The table also presents the ad valorem tax rate for industry 2. Note that the industry with the higher elasticity always has the higher ad valorem rate. As ϵ_1 rises, industry 1's rate rises relative to industry 2's, and eventually industry 2 receives a subsidy.¹³ This reverses another conventional wisdom that industries with higher elasticities should face lower tax rates. As mentioned in remark 3.1.2, this conventional wisdom is not so wise even in the perfectly competitive case.

Figure 2 shows the effect of various parameters. From proposition 3.1.4(a), any change that strengthens the Ramsey effect favors an inverse elasticities tax rule, while any change that boosts the producer markup favors a pro-elasticities rule. The top left panel shows the effect of changes in government

¹³The subsidy reflects, in part, the modest level of $g + n_1 F_1 + n_2 F_2$.

ϵ_1	1.05	1.25	1.50	1.75	2.00	2.25	2.50
P_1	1.183	1.149	1.127	1.115	1.108	1.103	1.100
p_1	1.157	1.129	1.105	1.089	1.077	1.068	1.061
$P_1 - p_1$	0.025	0.020	0.022	0.026	0.031	0.035	0.039
$t_1 = (P_1 - p_1)/p_1$	0.022	0.017	0.020	0.024	0.028	0.033	0.037
$t_2 = (P_2 - p_2)/p_2$	0.070	0.039	0.020	0.009	0.002	-0.002	-0.005

Table 1: Numerical results for the example's optimal tax equilibrium. Different columns correspond to different values for the elasticity parameter ϵ_1 . Other parameter values and the normalizations are held constant: $\epsilon_2 = 1.5$, $c_1 = c_2 = 1$, $n_1 = n_2 = 7$, $g + n_1 F_1 + n_2 F_2 = 0.05$, $p_0 = P_0 = 1$.

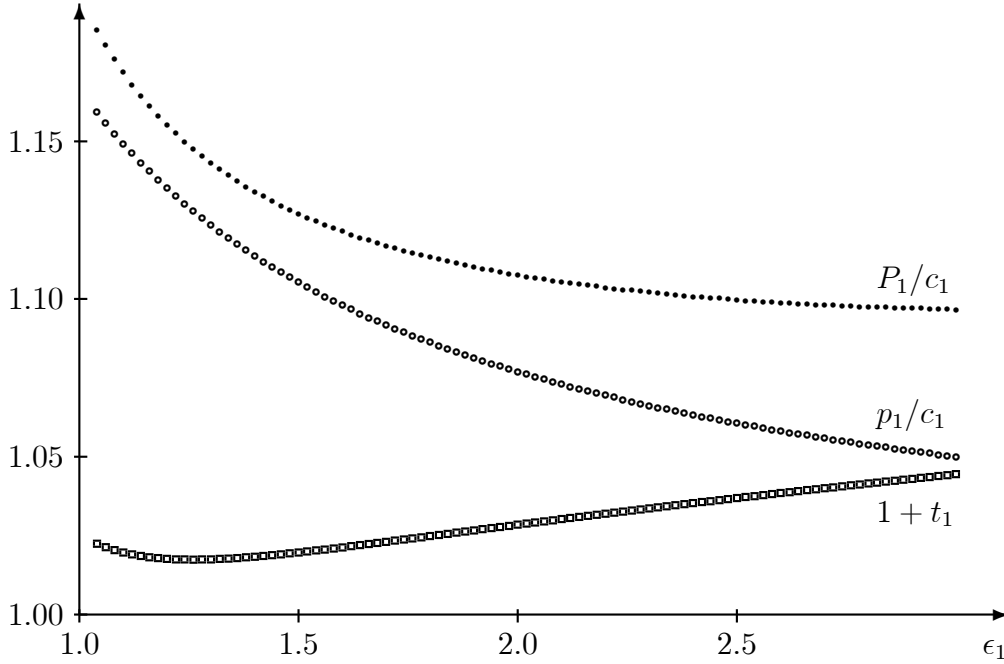


Figure 1: The top curve (dots) shows the Ramsey markup P_1/c_1 as a function of the own-price elasticity ϵ_1 . The middle curve (circles) shows the producer markup p_1/c_1 . The bottom curve (boxes) shows $1 + t_1$ where t_1 is the ad valorem tax rate. Parameter values and normalizations are the same as table 1.

spending, g . The middle curve in this panel reproduces the baseline and is labeled with $g = 0.02$ (hence $n_1 F_1 + n_2 F_2 = 0.03$). Larger values of g raise the need for revenue so the whole tax curve shifts up. Also, larger values of g increase the dependence on distortionary taxation so the Ramsey effect becomes more dominant and the tax rule moves in the direction of inverse elasticities.

The top right panel shows the effect of changes in market power in industry 1. This is achieved by changing the number of firms, n_1 , with no change in aggregate fixed costs. I.e., $n_1 F_1$ is kept constant so larger values of n_1 reduce the producer markup without adding to deadweight fixed costs. As $n_1 \rightarrow \infty$, the markup vanishes and the industry is, in effect, perfectly competitive. Since smaller n_1 boosts the markup, this favors a pro-elasticities tax rule as illustrated. The figure also shows that the whole curve shifts down for smaller n_1 . Recall that the tax is a combination of a corrective subsidy to neutralize the markup and a revenue raising component. With fewer firms, the markup is larger so the corrective subsidy is larger and this pulls down the tax.

The bottom left panel shows the effect of changes in marginal cost in industry 1. Changes in c_1 (from curve to curve) and changes in ϵ_1 (along each curve) interact in a particular way for this example. An increase in ϵ_1 not only raises the elasticity of demand for good 1, it also shifts out the demand curve since $Q_1 = \alpha_1 (P_1/P_0)^{-\epsilon_1} (P_2/P_0)^{1-\epsilon_2}$ and α_1 increases with ϵ_1 . So if c_1 is larger, an increase in ϵ_1 , other things equal, causes a larger increase in labor costs in industry 1. The government recognizes this cost and offsets it in part with policy to reduce demand: an increase in t_1 . I.e., a large c_1 favors a pro-elasticities rule.

The bottom right panel shows the effect of changes in ϵ_2 , the elasticity of demand in industry 2. Since the government internalizes cross price effects, the optimal choice for P_1 is influenced not only by ϵ_1 but also by ϵ_2 . In this particular example, when marginal costs equal one, P_1 is a function of $\epsilon_1 + \epsilon_2$. Hence, as ϵ_2 increases, ϵ_1 becomes proportionately less of an influence on P_1 , which works against the inverse elasticities rule. As the figure illustrates, larger values of ϵ_2 favor a pro-elasticities rule.

These exercises identify the way in which the optimal tax rule responds to some key parameters. For any other choice of functional forms, the results may differ to some extent, yet the same type of reasoning will still apply. One must compare those features that enhance the Ramsey effect (inverse elasticities) with those that enhance the producer markup (pro-elasticities).

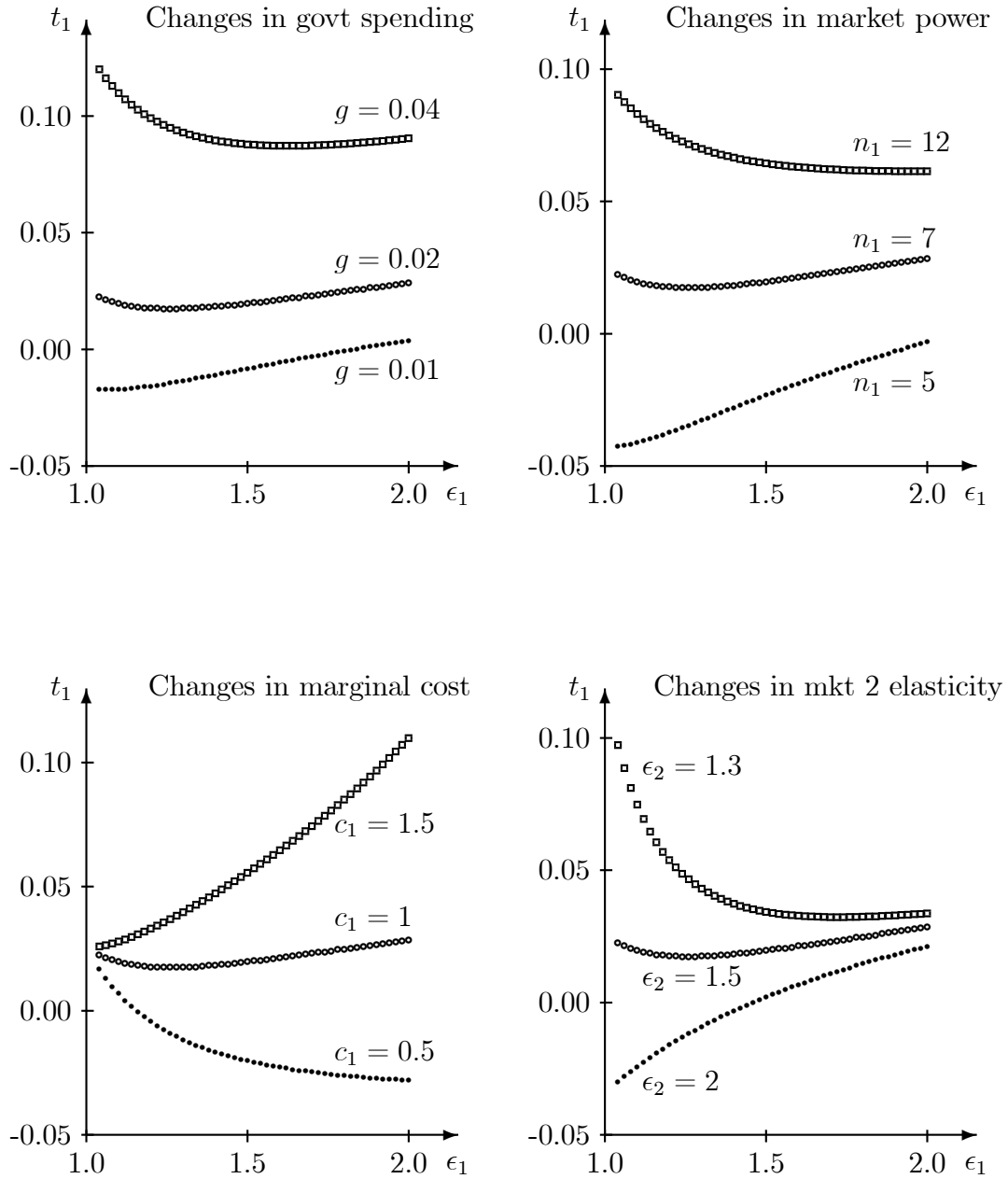


Figure 2: The optimal ad valorem tax rate t_1 as a function of the own-price elasticity ϵ_1 . In each panel, the middle curve reproduces the baseline. The other curves illustrate departures from the baseline, as indicated. Baseline parameters and normalizations are given in table 1.

5 Comparison of ad valorem and specific taxation

The theoretical analysis in section 3 overlooked a subtlety. At the end of section 2.2, one of the conditions for a symmetric Cournot equilibrium is

$$\epsilon_{ii} > 1/n_i \quad \text{for all } i. \quad (19)$$

This constraint was not imposed on the optimal tax problem. Consequently, the solution to that problem — the DM allocation — will not be implementable unless it satisfies (19).¹⁴ If this elasticity bound does not hold, the optimal tax problem is unlikely to have any solution with ad valorem taxes. Since the DM problem has no elasticity constraints built into it, this scenario cannot be ruled out a priori.

If specific taxes and subsidies are used rather than ad valorem, no bound is imposed on ϵ_{ii} , so the DM allocation can always be implemented. To demonstrate this, it is convenient to simplify the notation by dropping the industry subscript i and suppressing all but the essential variables. The firm's problem is to choose $p \geq 0$ to maximize

$$p \cdot [Q(P(p)) - \hat{Q}] - p_0 C(Q(P(p)) - \hat{Q})$$

where $P(p)$ is given by $P = (1 + t)p$ in the ad valorem case, and $P = p + t$ in the specific case. The first order condition is

$$Q - \hat{Q} + (p - p_0 C') \frac{dQ}{dP} \frac{dP}{dp} = 0.$$

Since $Q - \hat{Q} = Q/n$ in equilibrium, this yields

$$\frac{1}{n\epsilon} = (1 - p_0 C'/p) \frac{d \log P}{d \log p}.$$

In the ad valorem case, $d \log P / d \log p \equiv 1$, hence the bound in (19). In the specific case, $d \log P / d \log p = p/P$. Therefore,

$$\frac{1}{n\epsilon} = \frac{p - p_0 C'}{P} = \frac{P - t - p_0 C'}{P}.$$

This always has a solution for t , even if (19) does not hold. Though in that case, $t \leq -p_0 C'$ which could be a rather hefty subsidy.¹⁵ Thus, in some

¹⁴ If this condition fails, the formulas for $1 + t_i$ and p_i yield nonsense results. Specifically, if $\epsilon_{ii} = 1/n_i$ then $1 + t_i = 0$ and $p_i = P_i/(1 + t_i)$ is undefined, while if $\epsilon_{ii} < 1/n_i$ then $1 + t_i < 0$ and $p_i < 0$.

¹⁵ When (19) does not hold, p remains positive unlike the ad valorem case (footnote 14). This follows since $p = P/(n\epsilon) + p_0 C'$ which is positive if P is positive.

cases specific taxes and subsidies are preferred to ad valorem. This stands in contrast to Delipalla and Keen (1992), which extends a result due to Suits and Musgrave (1953), where ad valorem taxation is always preferred. The source of the difference in results is that their model does not include a tax on profits. This demonstrates that the optimal choice between two tax instruments may be affected by the set of other instruments that are already in use.

6 Conclusion

When firms are not perfectly competitive the familiar inverse elasticities rule for optimal tax rates is weakened and perhaps even reversed. The reason is that the tax rates are used in part as instruments to offset the adverse welfare effects from producer markups. Producer markups are higher in industries where demand is less elastic, so tax rates in these industries should be lowered to counteract the markups and thereby improve social welfare.

Two other results were obtained. First, if the government has a rich enough set of policy instruments then the adverse welfare effects from imperfect competition can be completely nullified. See also Myles (1996). Second, specific taxation may be preferred to ad valorem in some cases. This latter result differs from Delipalla and Keen (1992), and it demonstrates that the optimal choice between two instruments can depend on the entire array of policy instruments in use.

The paper used a model with identical consumers. If consumers are heterogeneous, the optimal tax policy continues to have a two-part interpretation as emphasized in the text. First, subsidize all of the imperfectly competitive industries to exactly offset the producer markups. Then raise revenue to cover the cost of the subsidies plus all other government operations by following the Diamond (1975) rule for a perfectly competitive economy with heterogeneous consumers. Since the first part of this decomposition subsidizes (lowers the tax rate) most where demand is least elastic, the effect of imperfect competition is to push optimal tax rates away from any inverse elasticities rule. The logic of this argument extends beyond the Cournot case to more general models, including those with imperfect competition in markets for intermediate goods (Judd, 2002, 2003).

The empirical evidence demonstrates that non-trivial markups are present throughout the US economy (e.g., Domowitz, Hubbard, and Petersen, 1988, Hall, 1988). This suggests that perfectly competitive Ramsey tax rules may be suboptimal. The challenge then is to devise an alternative policy that addresses the welfare costs of producer markups, and that still allows firms to recover their fixed costs. One of the messages from this paper is that a carefully designed tax system can accomplish this.

Appendix A Details of the agents' problems

Appendix A.1 Consumers

Consumers always choose an interior consumption point. Then no industry ever closes down. This is a convenient assumption since, with fixed costs of production, an industry closure would create a discontinuity. Formally, assumption 1(b) states that any indifference surface of U that has a non-empty intersection with the interior of the non-negative orthant is in fact contained entirely within the interior of the non-negative orthant. This, together with assumption 1(a) implies that $\ell(\mathbf{P}, M)$, $\mathbf{Q}(\mathbf{P}, M)$, and $V(\mathbf{P}, M)$ are all smooth functions on the strictly positive orthant (Mas-Colell, 1985, section 2.7). Since all consumers are identical, assumption 1(c) ensures that each industry faces a downward sloping demand curve.

Assumption 1.

- (a) U is defined on the non-negative orthant where it is a continuous function that does not attain a maximum. On the interior of the non-negative orthant, U is strictly quasi-concave and smooth (has derivatives of all orders). The gradient of U never vanishes. The determinant of the bordered Hessian matrix of U never vanishes.
- (b) If $(\ell, Q_1, \dots, Q_I) \gg \mathbf{0}$ and $0 \in \{\hat{\ell}, \hat{Q}_1, \dots, \hat{Q}_I\}$ then $U(\ell, Q_1, \dots, Q_I) > U(\hat{\ell}, \hat{Q}_1, \dots, \hat{Q}_I)$.
- (c) All goods ℓ, Q_1, \dots, Q_I are normal (positive income elasticity of demand) under U .

Appendix A.2 Firms

Assume the firms' cost functions are smooth and convex (for strictly positive output), and that marginal costs are strictly positive:

Assumption 2. For each i , $C_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ with $C_i(0) = 0$. On \mathbb{R}_{++} each C_i is smooth, strictly increasing, and convex.

From section 2.2, the profit function for a typical firm in industry i is

$$\pi_i(p_i; p_0, t_i, \hat{Q}_i, M, \mathbf{P}_{-i}) := p_i \cdot [Q_i(\mathbf{P}, M) - \hat{Q}_i] - p_0 C_i(Q_i(\mathbf{P}, M) - \hat{Q}_i).$$

The arguments of π_i must satisfy $0 < p_i \leq (1 + t_i)^{-1} P_i(\hat{Q}_i, M, \mathbf{P}_{-i})$ and $(p_0, 1 + t_i, \hat{Q}_i, M, \mathbf{P}_{-i}) \gg \mathbf{0}$. The upper bound on p_i is equivalent to $q_i \geq 0$. The lower bound on p_i is a strict inequality because demand might not even be defined when price equals zero. In any event, it is innocuous to restrict the firm from choosing $p_i = 0$ since that price generates no revenue.

Assumption 3. Consider π_i as a function of p_i on the following open interval: $0 < p_i < (1 + t_i)^{-1}P_i(\hat{Q}_i, M, \mathbf{P}_{-i})$. Assume that π_i is transformable into a strictly concave function by means of a differentiable bijection of the domain (p_i) and a differentiable, strictly increasing transformation of the range. (If π_i is already strictly concave then it satisfies this condition.)

Assumption 3 guarantees that any solution to $\partial\pi_i/\partial p_i = 0$ which yields $\pi_i > 0$ (and hence is preferred to the corner solution $q_i = 0$) uniquely solves the profit maximization problem (Ben-Tal, 1977).¹⁶ It is sufficient for π_i to be concave transformable only at the equilibrium values for $p_0, t_i, \hat{Q}_i, M, \mathbf{P}_{-i}$. Implicitly, this assumption imposes restrictions on the demand curves and hence the utility function. In general, these are restrictions on the third order derivatives of U since concave transformability of π_i has a characterization involving the second order derivatives of the demand functions (Ben-Tal, 1977). Whether or not assumption 3 holds in practice is an empirical question.

Recall from section 2.2 that the condition for a symmetric Cournot equilibrium is, for all $i \geq 1$,

$$Q_i(\mathbf{P}, M)/n_i + P_i \partial Q_i / \partial P_i - (1 + t_i) p_0 C'_i(Q_i(\mathbf{P}, M)/n_i) \cdot \partial Q_i / \partial P_i = 0. \quad (2)$$

Assumption 4. Let $(t_1, \dots, t_I) \gg -1$, $M > 0$, $P_0 > 0$, and $p_0 > 0$ be given. If (2) has a solution $(P_1, \dots, P_I) \gg 0$, then the solution is unique.

Solving (2) for consumer prices P_i is equivalent to solving for producer prices p_i since $P_i = (1 + t_i)p_i$. The existence of a unique solution to (2) is required only at the government's optimal choice of tax rates. Existence will be assured at that optimum since the government will choose the tax rates t_i to be consistent with (2) for a targeted set of consumer prices \mathbf{P} and lump sum income M .¹⁷ Assumption 4 requires the solution to be unique. This avoids the need to worry about coordination problems which would otherwise arise if there were multiple symmetric Cournot equilibria.

Appendix B Existence of an optimum

This appendix proves that there is a solution to the indirect utility maximization problem considered in section 3. Appendix C then establishes that the solution(s) must satisfy the first order conditions. Throughout, reference will be made to assumption 5 which is stated in appendix C.

To prove the existence of a solution, normalize prices using the unit simplex, $\Delta := \{(P_0, \dots, P_I) \geq 0 : \sum_{j=0}^I P_j = 1\}$.

¹⁶When the fixed costs $C_i(0^+)$ are not too large the solution will be interior.

¹⁷If the government behaved differently, or if tax rates were set exogenously, then existence could be a problem.

Inequality (9) (with $M = P_0 L$) defines the constraint set to consist of only those price vectors $\mathbf{P} \geq \mathbf{0}$ that generate technologically feasible allocations. It is convenient to replace the consumption set \mathbb{R}_+^{I+1} with $\{(\ell, \mathbf{Q}) \geq \mathbf{0} : \ell + \sum_{i=1}^I n_i C_i(Q_i/n_i) \leq L\}$. This is a compact convex set which is not empty under assumption 5(a). Since $g > 0$, every feasible allocation satisfies $\ell + \sum_{i=1}^I n_i C_i(Q_i/n_i) < L$. Thus, truncating the consumption set in this way does not alter the constraint set as defined by (9), nor does it affect the level of indirect utility on the constraint set. However, it does ensure that the demand functions are well-defined throughout Δ .

The constraint set consists of those points where the offer curve intersects the set of feasible allocations. (The offer curve is just the range of the vector of consumer demands as prices cover Δ .) This set could be empty if g is too large. Assumption 5(a) below rules out this possibility.

The existence of a maximum would be established if the demand functions were continuous. For then the objective function $V(\cdot)$ would be continuous, and the constraint set would be non-empty and compact. Unfortunately, there may be prices where demand is not continuous. From Debreu (1959), demand will be continuous at \mathbf{P} if the endowment is not a point of minimum wealth at those prices. DM satisfy this requirement for continuity by taking an interior endowment point. But here, only leisure is endowed in a positive quantity. So the endowment could be a point of minimum wealth if $P_0 = 0$. Otherwise, demand will be continuous.

It is impossible for any price vector with $P_0 = 0$ to maximize indirect utility subject to (9). To see this, note that there must be at least one $i \geq 1$ with $P_i > 0$ and for any such i , $Q_i(\mathbf{P}, P_0 L) = 0$ to satisfy the budget constraint, since $P_0 = 0$ implies that wealth is zero. The interiority assumption 1(b), and assumption 5(a) below rule out such a point as a maximizer.

Even though demand is not continuous throughout Δ , a maximum still exists. Let V^* be the supremum of indirect utility on the constraint set. Then there are prices \mathbf{P}^n in the constraint set such that $V(\mathbf{P}^n, P_0^n L) \uparrow V^*$. Let $\ell^n := \ell(\mathbf{P}^n, P_0^n L)$ and $\mathbf{Q}^n := \mathbf{Q}(\mathbf{P}^n, P_0^n L)$. Then $\{(\mathbf{P}^n, \ell^n, \mathbf{Q}^n)\}_{n=1}^\infty$ has a limit point $(\mathbf{P}^*, \ell^*, \mathbf{Q}^*)$ with $\mathbf{P}^* \in \Delta$ and (ℓ^*, \mathbf{Q}^*) in the compact consumption set. To save on notation, assume that the original sequence converges. Now if demand is continuous at \mathbf{P}^* then \mathbf{P}^* lies in the constraint set and $V(\mathbf{P}^*, P_0^* L) = V^*$, so \mathbf{P}^* is a maximizer. Otherwise, $P_0^* = 0$. I will prove that $P_0^* \neq 0$. First observe that the demand functions satisfy the budget constraint:

$$P_0^n \ell^n + \sum_{i=1}^I P_i^n Q_i^n \leq P_0^n L \quad n \geq 1.$$

Let n tend to infinity to get

$$P_0^* \ell^* + \sum_{i=1}^I P_i^* Q_i^* \leq P_0^* L.$$

Thus (ℓ^*, \mathbf{Q}^*) could have been purchased at prices \mathbf{P}^* . Therefore,

$$\begin{aligned} V(\mathbf{P}^*, P_0^* L) &\geq U(\ell^*, \mathbf{Q}^*) \\ &= \lim_{n \rightarrow \infty} U(\ell^n, \mathbf{Q}^n) \\ &= \lim_{n \rightarrow \infty} V(\mathbf{P}^n, P_0^n L) \\ &= V^*. \end{aligned}$$

On the other hand, if $P_0^* = 0$ then as discussed above there would be at least one i with $Q_i(\mathbf{P}^*, P_0^* L) = 0$ so $V(\mathbf{P}^*, P_0^* L)$ would be strictly less than V^* . This rules out the possibility that $P_0^* = 0$ and thereby establishes that a maximum exists.

Appendix C Necessity of the first order conditions

In order to show that the first order conditions (10) must hold at any maximum it must first be the case that demand is sufficiently smooth there to evaluate the necessary derivatives. This will be true if $\mathbf{P}^* \gg \mathbf{0}$. Furthermore, with $\mathbf{P}^* \gg \mathbf{0}$, (9) holds with equality (DM) and it is appropriate to take (10) as an equality, as written, rather than as a weak inequality. If preferences are strictly monotone then no price can be zero at the maximum. This is assumption 5(b). Without a constraint qualification the maximizer(s) might not satisfy (10). Following DM, assumption 5(c) rules out any tangency between the offer curve and the set of feasible allocations. Then \mathbf{P}^* must satisfy (10).

Assumption 5.

- (a) There exists $\mathbf{P} \in \Delta$ that satisfies $\ell(\mathbf{P}, P_0 L) > 0$, $Q_i(\mathbf{P}, P_0 L) > 0$ for $i \geq 1$, and

$$g + \ell(\mathbf{P}, P_0 L) + \sum_{i=1}^I n_i C_i(Q_i(\mathbf{P}, P_0 L)/n_i) \leq L.$$

- (b) Preferences are strictly monotone: $(\ell, \mathbf{Q}) \geq (\hat{\ell}, \hat{\mathbf{Q}})$ implies $U(\ell, \mathbf{Q}) \geq U(\hat{\ell}, \hat{\mathbf{Q}})$ with equality only if $(\ell, \mathbf{Q}) = (\hat{\ell}, \hat{\mathbf{Q}})$.

- (c) Suppose $g + \ell(\mathbf{P}, P_0 L) + \sum_{i=1}^I n_i C_i(Q_i(\mathbf{P}, P_0 L)/n_i) = L$ for some $\mathbf{P} \gg \mathbf{0}$. Then

$$\frac{\partial}{\partial P_j} \left[\ell(\mathbf{P}, P_0 L) + \sum_{i=1}^I n_i C_i(Q_i(\mathbf{P}, P_0 L)/n_i) \right] \neq 0 \quad j \geq 1.$$

Assumption 5(c) deserves a brief comment. Since $\partial V/\partial P_j < 0$ wherever $\mathbf{P} \gg \mathbf{0}$, the derivative in 5(c) must be non-positive at \mathbf{P}^* . Otherwise a feasible increase in utility would be available. Therefore, 5(c) requires that derivative to be strictly negative at \mathbf{P}^* .

Assumption 5 does not rule out the possible existence of additional solutions to (9) and (10) other than the global maximizers. These conditions are necessary, but in general they are not sufficient. This is a common problem in the optimal taxation literature — e.g., Atkinson and Stiglitz (1980, ch. 12), DM, Mirrlees (1986).

Appendix D Zero profits tax

Recall that the equilibrium conditions are (4) through (8). Solve (5) for $1 + t_i$, substitute this into (7), and use (4) to get

$$M = P_0 L + \sum_{i=1}^I p_0 (1 - t_i^\pi) \times \left[\frac{P_i Q_i(\mathbf{P}, M) C'_i(Q_i(\mathbf{P}, M)/n_i) \partial Q_i / \partial P_i}{Q_i(\mathbf{P}, M)/n_i + P_i \partial Q_i / \partial P_i} - n_i C_i(Q_i(\mathbf{P}, M)/n_i) \right]$$

where $\partial Q_i / \partial P_i$ is evaluated at (\mathbf{P}, M) . Assume that this equation has a unique solution $M = \mu(\mathbf{P}, p_0(\mathbf{1} - \mathbf{t}^\pi))$, defined on some domain.¹⁸ If $\mathbf{t}^\pi = \mathbf{1}$ then $\mu(\mathbf{P}, \mathbf{0}) = P_0 L$ as in section 3. Note that μ is homogeneous of degree one: $\mu(\lambda \mathbf{P}, \lambda p_0(\mathbf{1} - \mathbf{t}^\pi)) = \lambda \mu(\mathbf{P}, p_0(\mathbf{1} - \mathbf{t}^\pi))$.

Now substitute into the resource constraint (8) to get one condition that characterizes equilibrium:

$$g + \ell(\mathbf{P}, \mu) + \sum_{i=1}^I n_i C_i(Q_i(\mathbf{P}, \mu)/n_i) \leq L$$

where the arguments of μ have been suppressed. The government's problem is to maximize $V(\mathbf{P}, \mu)$ subject to the above constraint. If the profits tax is shut down as in Myles (1989), then $\mathbf{t}^\pi = \mathbf{0}$ and $p_0 = P_0$.¹⁹ The Lagrangian is

$$\mathcal{L} = V(\mathbf{P}, \mu) - \eta \left[\ell(\mathbf{P}, \mu) + \sum_{i=1}^I n_i C_i(Q_i(\mathbf{P}, \mu)/n_i) \right]$$

¹⁸If there were many households, each would have an equation describing its sources of income, and these equations would then be solved simultaneously. In this case, $Q_i(\mathbf{P}, \mathbf{M})$ would be a function of the vector of households' incomes.

¹⁹A 100 percent profits tax could be mimicked with $p_0/P_0 = 0$. Hence a restriction is necessary and the natural choice is to exempt labor from taxation: $p_0 = P_0$. See Myles (1989, pp. 96–97) for further discussion.

with μ evaluated at $(\mathbf{P}, P_0 \mathbf{1})$. Assuming differentiability, the first order condition for P_j is

$$\frac{\partial V}{\partial P_j} + \frac{\partial V}{\partial M} \frac{\partial \mu}{\partial P_j} = \eta \sum_{i=0}^I C'_i \left[\frac{\partial Q_i}{\partial P_j} + \frac{\partial Q_i}{\partial M} \frac{\partial \mu}{\partial P_j} \right] \quad j \geq 1 \quad (20)$$

where $Q_0 := \ell$ and $C'_0 := 1$. (Labor is transformable one-for-one into leisure.) By homogeneity, the first order condition for P_0 is redundant. Note that the first best is achieved when μ equals the consumers' expenditure function evaluated at the first best utility. Then $\partial \mu / \partial P_j = Q_j$ so the left hand side of (20) is zero while the expression in brackets on the right hand side is the Slutsky substitution term, and the equation is satisfied with C'_i proportional to P_i : marginal cost pricing.

Further analysis of the first order conditions could proceed as in section 3.1. But for the sake of concreteness, consider again the example from section 4, and modify it by shutting down the profits tax. Recall that the solution to the consumers' and firms' problems are

$$\begin{aligned} Q_1 &= \alpha_1 (P_1/P_0)^{-\epsilon_1} (P_2/P_0)^{1-\epsilon_2} \\ Q_2 &= \alpha_2 (P_1/P_0)^{1-\epsilon_1} (P_2/P_0)^{-\epsilon_2} \\ \ell &= M/P_0 - (\alpha_1 + \alpha_2) (P_1/P_0)^{1-\epsilon_1} (P_2/P_0)^{1-\epsilon_2} \\ V &= M/P_0 + (1 - \alpha_1 - \alpha_2) (P_1/P_0)^{1-\epsilon_1} (P_2/P_0)^{1-\epsilon_2} \\ p_i/p_0 &= n_i \epsilon_i c_i / (n_i \epsilon_i - 1) \\ \Pi_i &= (p_i - p_0 c_i) Q_i - p_0 n_i F_i = p_0 c_i Q_i / (n_i \epsilon_i - 1) - p_0 n_i F_i \end{aligned}$$

where Π_i is total profits in industry i . In equilibrium, the consumers' income is $M = P_0 L + (1 - t_1^\pi) \Pi_1 + (1 - t_2^\pi) \Pi_2$. When $t_i^\pi = 0$ for all i and $p_0 = P_0$,

$$M/P_0 = L - n_1 F_1 - n_2 F_2 + c_1 Q_1 / (n_1 \epsilon_1 - 1) + c_2 Q_2 / (n_2 \epsilon_2 - 1). \quad (21)$$

It is quite helpful here that demand for goods 1 and 2 is independent of income since (21) is then an explicit solution for M/P_0 , after substitution.

With this solution for income, the government's objective is

$$\begin{aligned} V &= (1 - \alpha_1 - \alpha_2) (P_1/P_0)^{1-\epsilon_1} (P_2/P_0)^{1-\epsilon_2} + \frac{c_1 \alpha_1}{n_1 \epsilon_1 - 1} (P_1/P_0)^{-\epsilon_1} (P_2/P_0)^{1-\epsilon_2} \\ &\quad + \frac{c_2 \alpha_2}{n_2 \epsilon_2 - 1} (P_1/P_0)^{1-\epsilon_1} (P_2/P_0)^{-\epsilon_2} + \text{constant} \end{aligned}$$

ϵ_1	1.05	1.25	1.50	1.75	2.00	2.25	2.50
P_1	1.195	1.179	1.159	1.142	1.129	1.119	1.110
p_1	1.157	1.129	1.105	1.089	1.077	1.068	1.061
$P_1 - p_1$	0.037	0.050	0.054	0.054	0.052	0.051	0.049
$(P_1 - p_1)/p_1$	0.032	0.044	0.049	0.049	0.049	0.048	0.047
$(P_2 - p_2)/p_2$	0.069	0.061	0.049	0.038	0.029	0.021	0.016

Table 2: Numerical results for the example's optimal tax equilibrium with no profits tax. Different columns correspond to different values for the elasticity parameter ϵ_1 . Other parameter values and the normalizations are held constant: $\epsilon_2 = 1.5$, $c_1 = c_2 = 1$, $n_1 = n_2 = 7$, $g = 0.02$, $p_0 = P_0 = 1$.

and its budget constraint (which is equivalent to the resource constraint) is

$$\begin{aligned}
 g &\leq t_1 p_1 Q_1 / P_0 + t_2 p_2 Q_2 / P_0 \\
 &= P_1 Q_1 / P_0 + P_2 Q_2 / P_0 - p_1 Q_1 / p_0 - p_2 Q_2 / p_0 \\
 &= (\alpha_1 + \alpha_2) (P_1 / P_0)^{1-\epsilon_1} (P_2 / P_0)^{1-\epsilon_2} - \frac{n_1 \epsilon_1 c_1 \alpha_1}{n_1 \epsilon_1 - 1} (P_1 / P_0)^{-\epsilon_1} (P_2 / P_0)^{1-\epsilon_2} \\
 &\quad - \frac{n_2 \epsilon_2 c_2 \alpha_2}{n_2 \epsilon_2 - 1} (P_1 / P_0)^{1-\epsilon_1} (P_2 / P_0)^{-\epsilon_2}
 \end{aligned}$$

where the second line uses $P_0 = p_0$. With the normalization $P_0 = 1$, the first order conditions for the constrained optimization problem can be manipulated to yield

$$\begin{aligned}
 &(n_2 \epsilon_2 - 1) [\alpha_1 + \alpha_2 + n_1 (1 - \alpha_2)] P_2 / c_2 \\
 &= (n_1 \epsilon_1 - 1) [\alpha_1 + \alpha_2 + n_2 (1 - \alpha_1)] P_1 / c_1 - n_1 \epsilon_1 + n_2 \epsilon_2.
 \end{aligned}$$

This equation can be solved for P_2 and substituted into the government's budget constraint (with equality) to get a single equation in P_1 . Then the optimal P_1 is the smallest solution that satisfies both $P_1 > 0$ and $P_2 > 0$.

Table 2 presents numerical results. Observe that the inverse elasticities rule is reversed for small values of ϵ_1 . For larger values, the tax on good 1 is relatively constant. As in table 1, the industry with the higher elasticity always has the higher ad valorem tax rate.

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